Conference 3

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Outline

- Hypothesis Testing and Confidence Interval
  - One-sample proportion test
  - Comparison of two proportions
    - Measure of association: Relative risk, risk different, Odds ratio
    - Chi-squared test
  - Pair-matched binary outcome

P1. A quality inspector for a wire manufacturer randomly selected 200 wire spools and found 11 of them to have unacceptable quality.

A. What is the 95% confidence interval for the true proportion of wires with defect?
B. Perform a hypothesis test to see if the proportion of unacceptable wires is different from 15%. What is the p value of this test? What can you conclude from it using significance level 0.05?
C. What are the conditions you need to check for problems B and C?

P1. A quality inspector for a wire manufacturer randomly selected 200 wire spools and found 11 of them to have unacceptable quality.

A. What is the 95% confidence interval for the true proportion of wires with defect?

$$\hat{\boldsymbol{p}} \pm z_{\alpha/2} \sqrt{\hat{\boldsymbol{p}}(\boldsymbol{1}-\hat{\boldsymbol{p}})/\boldsymbol{n}}$$

$\hat{p} = \frac{11}{200}$ =0.055

95% CI is $\left(0.055 - 1.96\sqrt{\frac{0.055(1-0.055)}{200}}, 0.055 + 1.96\sqrt{\frac{0.055(1-0.055)}{200}}\right)$

= (0.0234, 0.0866)

In R, prop.test() can be used to test proportions, but it is based on different test statistics and hence, the result can be slightly different from text book results.

> prop.test(x=11, n=200, conf.level=.95)

```
        1-sample proportions test with continuity correction

data:  11 out of 200, null probability 0.5
X-squared = 156.65, df = 1, p-value < 2.2e-16
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.02917549 0.09886691
sample estimates:
    p
0.055
```

P1. A quality inspector for a wire manufacturer randomly selected 200 wire spools and found 11 of them to have unacceptable quality.

B. Perform a hypothesis test to see if the proportion of unacceptable wires is different from 15%. What is the p value of this test? What can you conclude from it using significance level 0.05?

H_0:p=0.15

H_A:p ≠ 0.15

$$Z = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$$

$$Z = \frac{0.055-0.15}{\sqrt{0.15(1-0.15)/200}} = -3.76$$

|Z| = 3.76 > 1.96

```
> 2* (1-pnorm(3.76))   #P-value
```

Conclusion: P-value is 0.00017 and *with significance level 0.05, we have enough evidence to reject the null hypothesis that the proportion of unacceptable wires is different from 15%.*

```
> prop.test(x=11, n=200, p=.15, conf.level=.95)
```

```
        1-sample proportions test with continuity correction

data:  11 out of 200, null probability 0.15
X-squared = 13.422, df = 1, p-value = 0.0002487
alternative hypothesis: true p is not equal to 0.15
95 percent confidence interval:
 0.02917549 0.09886691
sample estimates:
    p
0.055
```

P1. A quality inspector for a wire manufacturer randomly selected 200 wire spools and found 11 of them to have unacceptable quality.

C. What are the conditions you need to check for problems B and C?

1. The sample observations are a simple random sample.
2. The conditions for a binomial distribution are satisfied.
3. Both number of successes and number of failures are at least 5.

When either number of success or number of failures is less than 5, binomial test can be used.

```
> binom.test(x=11, n=200, p=.15, conf.level=.95)

        Exact binomial test

data:  11 and 200
number of successes = 11, number of trials = 200, p-value = 4.066e-05
alternative hypothesis: true probability of success is not equal to 0.15
95 percent confidence interval:
 0.02777225 0.09627764
sample estimates:
probability of success
                 0.055
```

P2. A study looked at the effects of OC use on heart disease in women 40 to 44 years of age. The researchers found that among 5000 current OC users at baseline, 13 women developed a myocardial infarction (MI) over 3-year period, whereas among 10,000 never OC-users, 7 developed an MI over a 3-year period.

A. Assess the statistical significance of the results.

B. Provide a point estimate and 95% CI for the difference between the proportion of women who develop MI among OC users and the comparable proportion among non-OC users.

C. Provide a point estimate and a 95% CI for the relative risk of MI among OC users compared with non-OC users.

D. Estimate the OR in favor of MI for an OC user compared with a non-OC user. Provide a 95% CI for the OR.

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A. Assess the statistical significance of the results.

H_0: The proportion of MI among OC users is equal to the proportion of MI among non-OC users.

H_A: Two proportions are different.

	MI incidence		
	Yes	No	Total
OC-user	13	4987	5000
Non OC user	7	9993	10000

P2. A study looked at the effects of OC use on heart disease in women 40 to 44 years of age. The researchers found that among 5000 current OC users at baseline, 13 women developed a myocardial infarction (MI) over 3-year period, whereas among 10,000 never OC-users, 7 developed an MI over a 3-year period.

A. Assess the statistical significance of the results.

H_0: The proportion of MI among OC users is equal to the proportion of MI among non-OC users.

H_A: Two proportions are different.

	MI incidence		**Total**
	Yes	No	
OC-user	13 (20*5000/15000=6.67)	4987 (14980*5000/15000=4993.33)	5000
Non OC user	7 (20*10000/15000=13.33)	9993 (14980*10000/15000=9986.67)	10000
Total	20	14980	15000

$$\chi^2 = \frac{(13-6.67)^2}{6.67} + \frac{(7-13.33)^2}{13.33} + \frac{(4987-4993.33)^2}{4993.33} + \frac{(9993-9986.67)^2}{9986.67} = 9.025$$

$$P(\chi^2 > 9.025) = 0.0027$$

> 1-pchisq(9.025, df=1)=0.0027

```
> mat <- matrix(c(13, 7, 4987, 9993), ncol=2)
> chisq.test(mat)
```

```
> mat
     [,1] [,2]
[1,]   13 4987
[2,]    7 9993
> chisq.test(mat)

        Pearson's Chi-squared test with Yates' continuity correction

data:  mat
X-squared = 7.6665, df = 1, p-value = 0.005626
```

[Continuity correction makes the result a little bit different]

When any cell value is less than 5, the normal approximation to the binomial distribution is not valid. Fisher's exact test can be used.

```
> fisher.test(mat)

        Fisher's Exact Test for Count Data

data:  mat
p-value = 0.004002
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
  1.378818 11.021800
sample estimates:
odds ratio
  3.720772
```

B. Provide a point estimate and 95% CI for the difference between the proportion of women who develop MI among OC users and the comparable proportion among non-OC users.

$$\widehat{p_1} = \frac{13}{5000} = 0.0026$$
$$\widehat{p_2} = \frac{7}{10000} = 0.0007$$

Risk difference = $\widehat{p_1} - \widehat{p_2} = 0.0026 - 0.0007 = 0.0019$

$$95\%\ CI = 0.0019 \pm 1.96\sqrt{\frac{0.0026*0.9974}{5000} + \frac{0.0007*0.9993}{10000}}$$
$$= 0.0019 \pm 1.96*0.00077$$
$$= 0.0019 \pm 0.00150 = (0.0004, 0.0034)$$

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

C. Provide a point estimate and a 95% CI for the relative risk of MI among OC users compared with non-OC users.

$$\log(\hat{p}_1 / \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{q}_1}{n_1\hat{p}_1} + \frac{\hat{q}_2}{n_2\hat{p}_2}}$$

Relative risk = 0.0026/0.0007 = 3.714

95% CI ?

95% CI of Log RR = $\log(3.714) \pm 1.96\sqrt{\frac{0.9974}{5000*0.0026} + \frac{0.9993}{10000*0.0007}}$

= log(3.714) ±1.96 * 0.4685= 1.312 ±0.918

= (0.394, 2.23)

95% CI of RR = (exp(0.394), exp(2.23)) = (1.48, 9.3)

D. Estimate the OR in favor of MI for an OC user compared with a non-OC user. Provide a 95% CI for the OR.

$$\log\left(\frac{ad}{bc}\right) \pm z_{\alpha/2}\sqrt{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$$

OR = $\dfrac{\frac{0.0026}{0.9974}}{\frac{0.0007}{0.9993}} = 3.72$

95% CI for log OR = $\log(3.72) \pm 1.96\sqrt{\frac{1}{13}+\frac{1}{4987}+\frac{1}{7}+\frac{1}{9993}}$

= 1.314$\pm$ 1.96*0.469 = (0.394, 2.23)

95% CI = (exp(2.81), exp(2.23))= (1.48, 9.33)

P3. We want to compare two different chemotherapy regiments for breast cancer after mastectomy. The two treatment groups should be as comparable as possible on other prognostic factors. To accomplish this goal, a matched study is set up such that a random member of each matched pair gets treatment A (chemotherapy) perioperatively (within 1 week after mastectomy) and for an additional 6 months, whereas the other member gets treatment B (chemotherapy only perioperatively). The patients are assigned to pairs matched on age (within 5 years) and clinical condition. The patients are followed for 5 years, with survival as the outcome variable.

	Survive for 5 years	Die within 5 years
Treatment A	526	95
Treatment B	515	106

Table 1. 2 x 2 contigency table comparing treatments A and B

		Treatment B patient	
		Survive for 5 years	Die within 5 years
Treatment A patient	Survive for 5 years	510	16
	Die within 5 years	5	90

Table 2. 2 x 2 contigency table with matched pair as the sampling unit based on 621 matched pairs

A. Use an appropriate contigency table and perform a test to compare two treatments.
B. Estimate the OR relating type of treatment to 5-year mortality and get 95% CI.

A. Use an appropriate contigency table and perform a test to compare two treatments.

Since this is a matched pair data, we use the McNemar's test.

$$\chi^2 = \frac{(|16-5|-1)^2}{16+5} = 4.77$$
$$P(\chi^2 > 4.77) = 0.029.\ d.f = 1$$

- 1- pchisq(4.77, 1)

```
> mat <- matrix(c(510, 5, 16, 90), ncol=2)
> mat
     [,1] [,2]
[1,]  510   16
[2,]    5   90
> mcnemar.test(mat)

        McNemar's Chi-squared test with continuity correction

data:  mat
McNemar's chi-squared = 4.7619, df = 1, p-value = 0.0291
```

- When the number of discordant pairs is less than 20, the exact p-value can be calculated from a binomial distribution.
- H0: p=0.5 (p=proportion of type A discordant A out of discordant pairs).
- P-value =

$$2 \sum_{k=0}^{r} \binom{r+c}{k} \left(\frac{1}{2}\right)^{r+c} \quad if\ r < (r+c)/2$$

```
> binom.test(r, r+c, p=0.5)
```

```
> binom.test(5, 21, p=.5)

        Exact binomial test

data:  5 and 21
number of successes = 5, number of trials = 21, p-value = 0.0266
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.08217588 0.47165983
sample estimates:
probability of success
             0.2380952
```

```
> pbinom(5, 21, p=.5)
[1] 0.01330185
> 2*pbinom(5, 21, p=.5)
[1] 0.0266037
```

B. Estimate the OR relating type of treatment to 5-year mortality and get 95% CI.

OR = 16/5= 3.2

95% CI for Log OR = $\log(3.2) \pm 1.96\sqrt{\frac{1}{16}+\frac{1}{5}}$

$$= 1.16 \pm 1.96 * 0.5123$$

= 1.16 ±1.0042=(0.159, 2.167)

95% CI for OR = (exp(0.159), exp(2.167))

= (1.17, 8.74)

P4. A study was conducted to investigate the relationship between maternal smoking during pregnancy and the presence of congenital malformations. Among children who suffer from an abnormality other than Down's syndrome or an oral cleft, 32.8 percent have mothers who smoked during pregnancy. The researchers are interested to see if this proportion is homogeneous for children with various types of defects. If the true population proportion of children with an oral cleft whose mothers smoked is as low as 0.25, you want to have 90 percent power to reject the null hypothesis. If you are conducting a two-sided test at the 0.01 level of significance, how large a sample would be required?

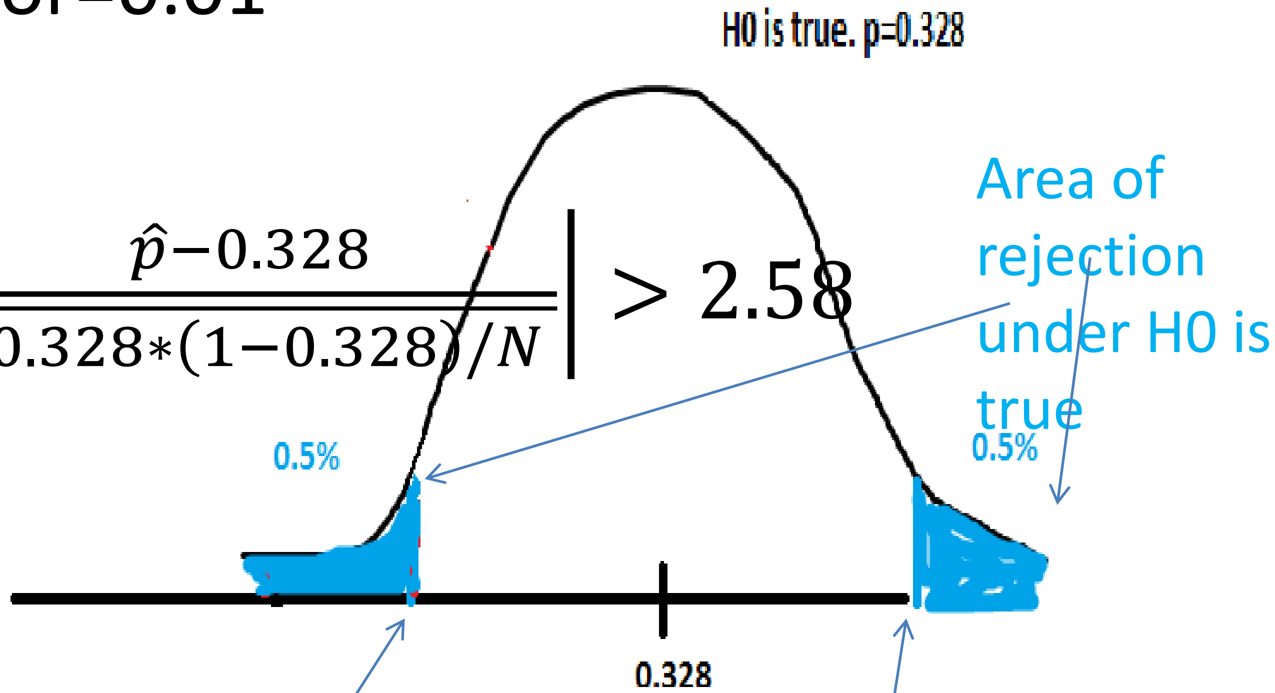
Type I error:

Assume the null hypothesis is true.

Type I error=0.01

P0=0.328

Z-test: $\left|\dfrac{\hat{p}-0.328}{\sqrt{0.328*(1-0.328)/N}}\right| > 2.58$



$$\hat{p} < 0.328 - 2.58\sqrt{.328 * (1-0.328)/N}$$

$$\hat{p} > 0.328 + 2.58\sqrt{.328 * (1-0.328)/N}$$

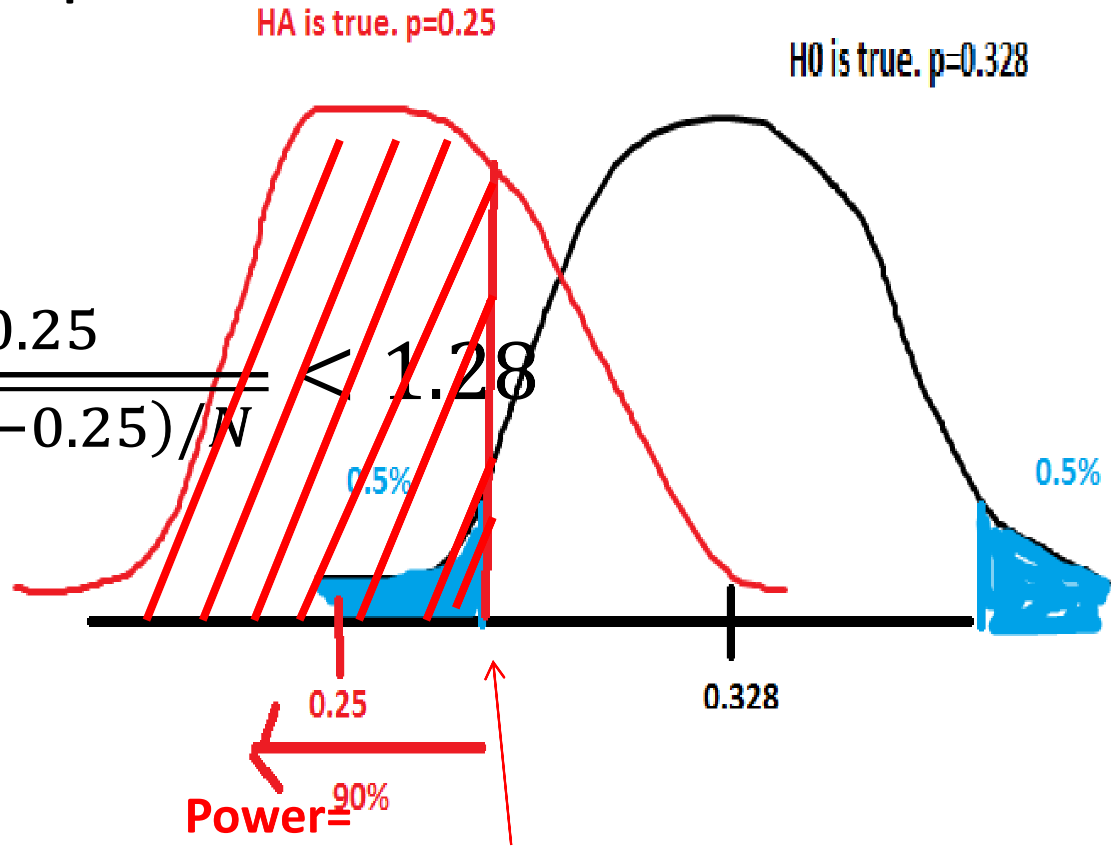
Type II error:

Assume true proportion is 0.25

P1=0.25

Power=90%

Z-test: $\frac{\hat{p}-0.25}{\sqrt{0.25*(1-0.25)/N}} < 1.28$



$$\hat{p} < 0.25 + 1.28\sqrt{.25 * (1 - 0.25)/N}$$

Equating

$$0.328 - 2.58\sqrt{.328 * (1 - 0.328)/N}$$
$$= 0.25 + 1.28\sqrt{.25 * (1 - 0.25)/N}$$

N=512.3

➔ Requires 513 samples